Tutorial 5.

Preliminary:

$(Ia)_{\overline{n}|i} = \frac{\ddot{a}_{\overline{n}|i} - nv^n}{i}$,

$(Ia)_{\overline{\infty}|i} = \frac{1}{i^2} + \frac{1}{i}$

Increasing Annuities: $(Ia)_{\overline{n}|i} = X$.

Present-Value.

$\underline{(I\ddot{a})_{\overline{n}|i}} = v + 2v^2 + \cdots + nv^n$, then $(1+i)X = 1 + 2v + \cdots + nv^{n-1}$.

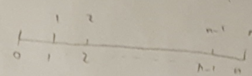
$iX = 1 + v + \cdots + v^{n-1} - nv^n$. $X = \frac{1 + \cdots + v^{n-1} - nv^n}{i} = \frac{\ddot{a}_{\overline{n}|i} - nv^n}{i} = (Ia)_{\overline{n}|i}$

perpetuity:

when $n \to \infty$, $(Ia)_{\overline{\infty}|i} = \lim_{n\to\infty} \frac{\ddot{a}_{\overline{n}|i} - nv^n}{i} = \frac{1}{id}$. because $\lim_{n\to\infty} \ddot{a}_{\overline{n}|i} = \frac{1}{d}$, $\lim_{n\to\infty} nv^n = 0$.

$= \frac{1}{i^2} + \frac{1}{i}$

A-V: $(Is)_{\overline{n}|i} = \frac{\ddot{s}_{\overline{n}|i} - n}{i}$



$X = (1+i)^{n-1} + 2(1+i)^{n-1} + \cdots + (n-1)(1+i) + n$

$(1+i)X = (1+i)^n + 2(1+i)^{n-1} + \cdots + (n-1)(1+i)^2 + n(1+i)$

$iX = (1+i)^n + (1+i)^{n-1} + \cdots + (1+i) - n$

$X = \frac{\ddot{s}_{\overline{n}|i} - n}{i}$

Decreasing Annuities:

$(Da)_{\overline{n}|i} = nv + (n-1)v^2 + \cdots + v^n = \frac{n - a_{\overline{n}|i}}{i}$.

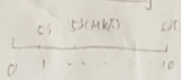
$(Ds)_{\overline{n}|i} = \frac{n(1+i)^n - s_{\overline{n}|}}{i} = (Da)_{\overline{n}|i} \cdot (1+i)^n$.

Exercise:

2.3.3.

Jeff: $X = 30\, a_{\overline{\infty}|k\%} = \frac{30}{k\%}$. "perpetuity-immediate".

Jason: "~~increasing~~"



$X = 53v + 53\cdot(1+k\%)v^2 + \cdots + 53(1+k\%)^9 v^{10}$

$= 53v\,(1 + (1+k\%)v + \cdots + (1+k\%)^9 v^9)$ $\cancel{= 53v \frac{1-(1+k\%)^{10}v^{10}}{1-(1+k\%)v}}$

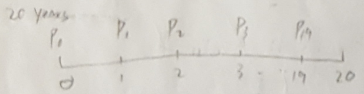
since $v = \frac{1}{1+k\%}$, then $(1+k\%)v = 1$.

hence, $X = 53v \cdot 10 = \frac{530}{1+k\%}$. $\frac{530}{1+k\%} = \frac{30}{k\%}$ $\Rightarrow$ $k\% = 6\%$.

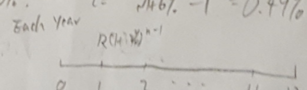
2.3.5.

$j = 6\%$ monthly rate $(1+i)^{12} = 1+6\%$. $\Rightarrow$ $i = \sqrt[12]{1+6\%} - 1 = 0.49\%$.

20 years



Each year



$\underline{P_n = R \cdot 1.032^n \cdot a_{\overline{12}|i}}$

$100{,}000 = P_0 + P_1 v_j + P_2 v_j^2 + \cdots + P_{19} v_j^{19} = R \cdot 1.032^0 \cdot a_{\overline{12}|i} + \cdots R \cdot 1.032^{19} \cdot a_{\overline{12}|i} \cdot v_j^{19}$

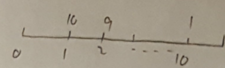
$= R\, a_{\overline{12}|i} \cdot (1 + 1.032 \cdot v_j + \cdots + 1.032^{19} v_j^{19}) = R \cdot \frac{1 - v_i^{12}}{i} \cdot \frac{1 - (\frac{1.032}{1.06})^{20}}{1 - \frac{1.032}{1.06}}$

$\Rightarrow R = 547.9$.

2.3.18.

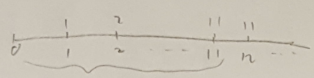
Present Value for Annuity 1.

$$(Da)_{\overline{10}|i} = \frac{10 - a_{\overline{10}|i}}{i}$$

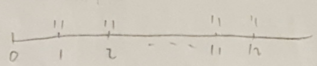


"$(Da)_{\overline{10}|i}$"

Present Value for Annuity 2.



The difference.



"$11a_{\overline{\infty}|i}$"

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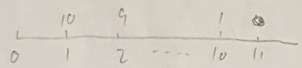

"$(Da)_{\overline{10}|i}$."

$$\Rightarrow \quad 11a_{\overline{\infty}|i} - (Da)_{\overline{10}|i} = 2(Da)_{\overline{10}|i}$$

$$11a_{\overline{\infty}|i} = 3(Da)_{\overline{10}|i}$$

$$\frac{11}{i} = 3 \cdot \frac{10 - a_{\overline{10}|i}}{i}$$

$$\Rightarrow a_{\overline{10}|i} = \frac{19}{3} \Rightarrow i = 0.093.$$

$$(Da)_{\overline{10}|i} = \frac{10 - \frac{19}{3}}{0.093} = 39.4.$$